Шамін Денис. Домашня робота №6

6.1 6.2 6.3 6.4 6.8 6.10 6.19

№6.1

а) $P\{\varphi(\text{двох}) = 1\} = P\{\xi = 1; \eta = 1\} = 1/36$

$P\{\varphi = 2\} = P\{(\xi = 1; \eta = 2) \cup (\xi = 2; \eta = 1) \cup$

$\cup (\xi = \eta = 2)\} =$ (сума ймовірності несумісних подій

$P\{\varphi = 3\} = P\{(\xi = 1; \eta = 3) \cup (\xi = 2; \eta = 3) \cup (\xi = 3;$

$\eta = 1) \cup (\xi = 3; \eta = 2) \cup (\xi = \eta = 3)\} = \frac{5}{36}$.

таблиця випадкових величин φ:

φ	1	2	3	4	5	6
p_k	$1/36$	$3/36$	$5/36$	$7/36$	$9/36$	$11/36$

$\sum_{k=1}^{6} p_k = 1$

г) $P\{\min(\xi, \eta) \le 1;\ \max(\xi, \eta) \geq 5\} = P\{\min(\xi, \eta) = 1;$

$(\max(\xi, \eta) = 5$ або $6) = P\{(\xi = 1, \eta = 5) \cup (\xi = 1; \eta = 6) \cup$

$\cup (\xi = 6; \eta = 1)\} = 4/36$.

№6.4

а) $P\{|\xi| < 1; \eta > 0\} = P\{(\xi, \eta) \in D_1\} = \int_{-1}^{1} dx \int_{0}^{+\infty} p(x,y)\, dy$

б) $P\{\xi < \eta\} = P\{(\xi, \eta) \in D_2\} = \int_{-\infty}^{+\infty} dx \int_{x}^{+\infty} p(x,y)\, dy$.

г) $P\{[\xi] = [\eta]\} = P\{(\xi, \eta) \in D_3\} = \sum_{k=-\infty}^{\infty} \int_{k}^{k+1} dx \int_{k}^{k+1} p(x,y)\, dy$

в) $P\{\xi < x\} = P\{(\xi, \eta) \in D_4\} = \int_{-\infty}^{x} dx \int_{x}^{+\infty} p(x,y)\, dy$.

№6.2

$P\{\xi = x_i\} = p_i$;

$P\{\eta = x_j\} = q_j$

$P = \{\xi = \eta\} - ?$

$$P\{(\xi = \eta = x_1) \cup (\xi = \eta = x_2) \cup \ldots\} = \sum_{i=1}^{\infty} P(\xi = \eta = x_i) = \sum_{i=1}^{\infty} p_i q_i$$

№6.3

$\zeta = \xi + \eta \Rightarrow z = x + y$

$\zeta = P(\xi + \eta < z) = \iint\limits_D f(x,y)\,dx\,dy$; $D = \{(x;y) : x + y < z\}$

$\zeta(z) = P(Z < z) = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{z-x} f(x,y)\,dy$

$\zeta'(z) = \int_{-\infty}^{+\infty} f(x, z-x)\,dx$; $\Rightarrow f(z) = \int_{-\infty}^{\infty} f(x, z-x)\,dx$

щільність розподілу: $f(z) = \int_{-\infty}^{+\infty} f_1(x) f_2(z-x)\,dx$.

№6.8

$$f(x,y) = \begin{cases} \dfrac{3y^2}{\pi(1+x^2)} & y \in [0,1] \\ 0, & y \notin [0,1] \end{cases}$$

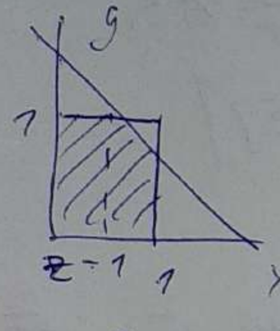
$P\{\xi\eta < 1\} - ?$

$f_\xi(x) = \int_{-\infty}^{+\infty} f_\zeta(x,y)\,dy = \int_0^1 \frac{3y^2}{\pi(1+x^2)}\,dy = \frac{1}{\pi + \pi x^2}$ $[0,1) \in y$

$f_\eta(y) = \int_{-\infty}^{+\infty} f_\zeta(x,y)\,dx = \int_{-\infty}^{+\infty} \frac{3y^2}{\pi(1+x^2)}\,dx = \frac{3y^2}{4}$

ξ, η залежні

(N6.10)



$y = z - x$

$(z > 1)$

$z \in (1; 2)$

$$\iint\limits_D p(x,y)\,dx\,dy = \int_0^z dx \int_0^{z+1} (x+y)\,dy = \int_0^z \left(x(z-x) + \frac{(z-x)^2}{2}\right)dx =$$

$$= \left(\frac{x\cdot z^2 - x^2\cdot z + \frac{x^3}{3}}{2} - \frac{2x^3 - 3x^2 z}{6}\right)\Big|_0^z =$$

$$= \frac{z^3 \cancel{z} - z^3 + \frac{z^3}{3}}{2} - \frac{2z^3 - 3z^3}{6} = \frac{z^3}{6} + \frac{z^3}{6} = \frac{2z^3}{6} = \frac{z^3}{3}$$

при $z \in (0; 1)$

$$\iint\limits_D p(x,y)\,dx\,dy = \int_0^{z-1} dx \int_0^1 (x+y)\,dy + \int_{z-1}^1 dx \int_0^{z-1} (x+y)\,dy$$

$$P_Y(z) = \begin{cases} z^2, & z \in (0; 1) \\ z(2-z) & z \notin (1; 2) \end{cases}$$